EFFECT OF VARIABILITY OF THE PHYSICAL PARAMETERS OF A GRAVITATIONAL LIQUID FILM ON ITS THICKNESS AND HEAT TRANSFER DURING LAMINAR FLOW

G. I. Gimbutis and I. Yu. Reklaitis

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A method and results are presented in regard to theoretical calculation of the heat transfer and thickness of a gravitational film of liquid with allowance for variability of its physical parameters.

The gravitational liquid film sprayed on the heat-exchange surface in film-type heat exchangers often has a high viscosity. In this instance, the flow regime of the film is usually laminar or wavelike, and heat transfer is distinguished by relatively low heat-transfer coefficients. This situation encourages the development of large temperature gradients across the film and substantial changes in its physical parameters. These changes must be taken into account during engineering calculations of heat transfer and film thickness.

In the laminar flow of a gravitational film, the above problem can be solved theoretically. It is most easily solved in the case of a stabilized flow and heat transfer.

The following can be written for the shear stress in the stabilized laminar flow of a gravitational liquid film

$$\tau = g\int\limits_{y}^{\delta}(\rho-\rho_g)\,dy = \rho\nu\frac{dw}{dy}. \tag{1}$$

Then the relative shear stress in the film

$$\frac{\tau}{\tau_c} = 1-\frac{\int\limits_0^{\eta}\frac{\rho-\rho_g}{\rho_f}\,d\eta}{\int\limits_0^{\eta_\delta}\frac{\rho-\rho_g}{\rho_f}\,d\eta} \tag{2}$$

and the dimensionless velocity field

$$\varphi = \int\limits_0^{\eta}\frac{\tau/\tau_w}{\frac{\rho\nu}{\rho_f\nu_f}}\,d\eta. \tag{3}$$

For the heat flux in the film we can write

$$q = -\lambda\frac{\partial T}{\partial y} = -\frac{\lambda v^*}{\nu_f}\frac{\partial T}{\partial\eta} \tag{4}$$

with the boundary conditions $y = 0$, $q = q_w$, $T = T_w$, $y = \delta$, $q = 0$. Integration of this equation leads to the following expression for the temperature field:

$$T = T_w - \frac{q_w\nu_f}{\lambda_f v^*}\psi, \tag{5}$$

where

$$\psi = \int\limits_0^{\eta}\frac{q/q_w}{\lambda/\lambda_f}\,d\eta. \tag{6}$$

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The heat-flux distribution across the film can be found from the energy equation

$$\rho w c \frac{\partial T}{\partial x} + \frac{\partial q}{\partial y} = 0. \tag{7}$$

With stablized heat transfer, the boundary conditions $y = 0$, $q = q_w$; $y = \delta$, $q = 0$. The energy equation is solved most easily with the wall boundary conditions q_w = const and T_w = const. According to [1, 2], it can be assumed with sufficient accuracy that with q_w = const

$$\frac{\partial T}{\partial x} = \frac{dT_f}{dx}, \tag{8}$$

while with T_w = const

$$\frac{\partial T}{\partial x} = \frac{T_w - T}{T_w - T_f} \frac{dT_f}{dx}. \tag{9}$$

The the solution of Eq. (7) leads to the following expression for the heat-flux distribution across the film

$$\frac{q}{q_w} = 1 - \frac{\int\limits_0^{\eta} \frac{\rho c}{\rho_f c_f} \varphi d\eta}{\int\limits_0^{\eta_\delta} \frac{\rho c}{\rho_f c_f} \varphi d\eta} \tag{10}$$

with q_w = const and

$$\frac{q}{q_w} = 1 - \frac{\int\limits_0^{\eta} \frac{\rho c}{\rho_f c_f} \varphi \psi d\eta}{\int\limits_0^{\eta_\delta} \frac{\rho c}{\rho_f c_f} \varphi \psi d\eta} \tag{11}$$

with T_w = const.

Let us divide Eq. (5) by the temperature head $T_w - T_f$ and consider that, in accordance with (1):

$$v^* = \left(\frac{\tau_w}{\rho_f}\right)^{1/2} = \left(g\nu_f \int\limits_0^{\eta_\delta} \frac{\rho - \rho_g}{\rho_f} d\eta\right)^{1/3}. \tag{12}$$

We will also allow that $q_w(T_w - T_f) = \alpha$. Then after some simple algebraic transformations, we obtain the following expression for the relative temperature field in the film:

$$\frac{T_w - T}{T_w - T_f} = \frac{\mathrm{Nu}_{Mf} \psi}{\left(\int\limits_0^{\eta_\delta} \frac{\rho - \rho_g}{\rho_f - \rho_g} d\eta\right)^{1/3}}. \tag{13}$$

It is obvious that

$$\frac{\int\limits_0^{\eta_\delta} (T_w - T) c\rho\varphi d\eta}{(T_w - T_f) \int\limits_0^{\eta_\delta} c\rho\varphi d\eta} = 1. \tag{14}$$

Then, with allowance for this, we find from Eq. (13) that

$$\mathrm{Nu_{mf}} = \frac{\left(\int\limits_0^{\eta_\delta} \frac{\rho c}{\rho_f c_f}\varphi d\eta\right)\left(\int\limits_0^{\eta_\delta} \frac{\rho-\rho_g}{\rho_f-\rho_g}\, d\eta\right)^{1/3}}{\int\limits_0^{\eta_\delta} \frac{\rho c}{\rho_f c_f}\varphi\psi d\eta}. \tag{15}$$

The spray density with variable physical parameters in the film

$$\Gamma = \int\limits_0^{\delta} w\rho dy \tag{16}$$

and, accordingly,

$$\mathrm{Re}_f = 4\int\limits_0^{\eta_\delta} \frac{\rho}{\rho_f}\varphi d\eta. \tag{17}$$

System of equations (2), (3), (6), and (10) or (11), (13), (15), and (17) makes it possible to determine the functions $\eta_\delta = f(\mathrm{Re})$ and $\mathrm{Nu_m} = f(\mathrm{Re})$ with allowance for the **variability** of the physical parameters of the liquid.

With constant physical parameters, the above equations are integrated analytically and we obtain familiar relations to calculate the film thickness (friction) and heat transfer:

$$\eta_\delta = \left(\frac{3}{4}\mathrm{Re}\right)^{1/2}, \tag{18}$$

$$\delta = \left(\frac{3}{4}\frac{\nu^2}{g}\frac{\rho}{\rho-\rho_g}\mathrm{Re}\right)^{1/3}, \tag{19}$$

$$\mathrm{Nu_m} = 2.27\,\mathrm{Re}^{-1/3} \text{ at } q_w = \text{const}, \tag{20}$$

$$\mathrm{Nu_m} = 2.07\,\mathrm{Re}^{-1/3} \text{ at } T_w = \text{const}. \tag{21}$$

As is known, the equivalent diameter of a planar film $d = 4\delta$. Allowing for this, we can use Eq. (19), (20), and (21) to obtain the following expressions:

$$\mathrm{Nu} = 8.24 \text{ at } q_w = \text{const}, \tag{22}$$

$$\mathrm{Nu} = 7.5 \text{ at } T_w = \text{const}. \tag{23}$$

The Nu number has the same values as in the case of **stabilized** heat transfer in a planar channel [1], since the film can be regarded as half of the flow in a planar channel.

With variable physical parameters, the problem is solved only by numerical methods. We calculated the thickness of the film (friction) and heat transfer for films of water, transformer and compressor oils, glycerol, and fuel oil M 100, using the method of successive approximations and employing tabulated data on physical parameters. The physical parameters of water and glycerol were taken from [3], while [4] and [5], respectively, were the sources for the compressor oil and fuel oil. The physical parameters of the transformer oil is shown in Table 1. The results of the calculations are shown in Fig. 1 and 2.

TABLE 1. Physical Parameters of the Transformer Oil

T, °C	ρ, kg/m³	λ, W(m·K)	$\nu\cdot10^6$, m²/sec	Pr
10	883	0,1165	46,0	560
20	877	0,1155	27,8	355
30	871	0,1140	17,2	233
40	865	0,1130	11,5	165
50	860	0,1120	8,2	123
60	854	0,1110	6,2	95
70	850	0,1100	4,5	78

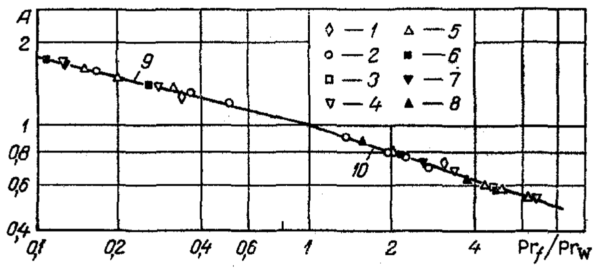

Fig. 1

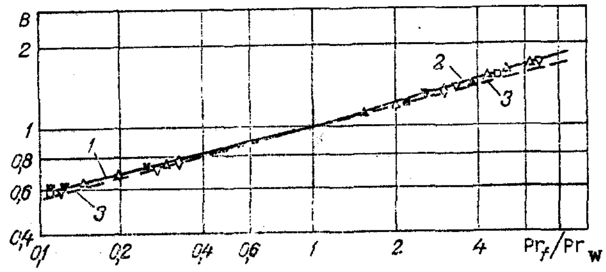

Fig. 2

Fig. 1. Effect of variability of the physical parameters of the film on its thickness: 1, 2, 3, 4, 5) films of water, transformer oil, compressor oil, glycerol, and fuel oil at q_w = const; 6, 7, 8) film of compressor oil, glycerol, and fuel oil at T_w = const; 9, 10) approximating straight lines with m = 0.25 and 0.32, respectively, $A = \eta_\delta/\eta_{\delta_0}$.

Fig. 2. Effect of variability of the physical parameters of the film on heat transfer: 1, 2, 3) approximating straight lines for n = 0.23, 0.28, and 0.25, respectively. The notation is the same as in Fig. 1; $B = Nu_{mf}/Nu_{mo}$.

The change in film thickness with the onset of nonisothermality is related mainly to a change in viscosity. However, the latter depends on the temperature head in the film, which in turn is dependent on the thermal parameters of the liquid. It is therefore best to use the ratio Pr_f/Pr_w to account for the effect of variability of the physical parameters on film thickness.

It can be seen from Figs. 1 and 2 that despite the quite different temperature dependencies of the physical parameters of the liquids used and the different boundary conditions on the wall, the calculated data is satisfactorily generalized by the equations

$$\frac{\eta_\delta}{\eta_{\delta 0}} = \left(\frac{Pr_f}{Pr_w}\right)^{-m}, \tag{24}$$

$$\frac{Nu_m}{Nu_m} = \left(\frac{Pr_f}{Pr_w}\right)^{n}. \tag{25}$$

With heating of the film, $m \simeq 0.32$, $n \simeq 0.28$, while with cooling, $m \simeq 0.25$, $n \simeq 0.23$. Since the index n is only slightly dependent on the direction of the heat flow, it can be assumed constant and equal to 0.25.

Thus, with allowance for Eqs. (18) and (24), the relation for the thickness of the film (friction) takes the following form:

$$\eta_\delta = \left(\frac{3}{4}\,Re_f\right)^{1/2} \left(\frac{Pr_w}{Pr_f}\right)^{m}. \tag{26}$$

The density of the liquid changes relatively little with temperature. Then, in accordance with Eq. (12):

$$v^* \simeq \left[g\nu_f\eta_\delta\left(1 - \frac{\rho_g}{\rho_f}\right)\right]^{1/3} \tag{27}$$

and

$$\eta_\delta = \frac{v^*\delta}{\nu_f} = \frac{\delta^{3/2}}{\nu_f}\left(g\,\frac{\rho_f - \rho_g}{\rho_f}\right)^{1/2}. \tag{28}$$

Thus:

$$\frac{\delta}{\delta_0} \simeq \left(\frac{\eta_\delta}{\eta_{\delta 0}}\right)^{2/3}. \tag{29}$$

Then, with allowance for (19) and (24), the thickness of a nonisothermal film:

$$\delta \simeq \left(\frac{3}{4}\,\frac{\nu_f^2}{g}\,\frac{\rho_f}{\rho_f - \rho_g}\,Re_f\right)^{1/3} \left(\frac{Pr_w}{Pr_f}\right)^{\frac{2}{3}m}. \tag{30}$$

With allowance for variability of the physical parameters, the relations for heat transfer take the following form in accordance with Eqs. (20)-(23) and (25)

$$\mathrm{Nu}_{mf} = 2.27\,\mathrm{Re}_f^{-1/3}\left(\frac{\mathrm{Pr}_f}{\mathrm{Pr}_w}\right)^{1/4} \quad (31)$$

or

$$\mathrm{Nu}_f = 8.24\left(\frac{\mathrm{Pr}_f}{\mathrm{Pr}_w}\right)^{1/4} \quad (32)$$

with q_w = const;

$$\mathrm{Nu}_{mf} = 2.07\,\mathrm{Re}_f^{-1/3}\left(\frac{\mathrm{Pr}_f}{\mathrm{Pr}_w}\right)^{1/4} \quad (33)$$

or

$$\mathrm{Nu}_f = 7.5\left(\frac{\mathrm{Pr}_f}{\mathrm{Pr}_w}\right)^{1/4} \quad (34)$$

with T_w = const. In both cases, the equivalent diameter in the Nu numbers is determined in the same manner as for an isothermal film:

$$d = 4\delta = 4\left(\frac{3}{4}\,\frac{\nu_f^2}{g}\,\frac{\rho_f}{\rho_f - \rho_g}\,\mathrm{Re}_f\right)^{1/3}. \quad (35)$$

In conclusion, we should note that a similar problem was solved in [6, 7]. However, these studies considered only the change in viscosity and used a model liquid for which the function $\nu = f(T)$ was approximated by a hyperbola.

NOTATION

$\mathrm{Nu}_M = \frac{\alpha}{\lambda}\left(\frac{\nu^2}{g}\,\frac{\rho}{\rho - \rho g}\right)^{1/3}$; $\mathrm{Nu} = \frac{\alpha d}{\lambda}$; $\mathrm{Re} = \frac{4\Gamma}{\rho\nu}$; $\mathrm{Pr} = c\rho\nu/\lambda$; $\eta = v^* y/\nu_f$ dimensionless distance from wall; $\eta_\delta = v^*\delta/\nu_f$, dimensionless thickness of film; $\varphi = w/v^*$, dimensionless velocity; τ_w, shear stress on the wall; $v^* = (\tau_w/\rho_f)^{1/2}$, dynamic velocity; $d = 4\delta$, equivalent diameter of film; τ, shear stress; α, local heat-transfer coefficient; λ, thermal conductivity of liquid; c, specific heat of liquid; ν, kinematic viscosity of liquid; ρ, density of liquid; ρ_g, density of gas; Γ, spray density; q, heat flux across film; T, temperature of liquid; w, local velocity of film; δ, film thickness; g, acceleration due to gravity; x, coordinate parallel to wall; y, coordinate perpendicular to wall. Indices: w, physical parameters of liquid taken at wall temperature; f, same taken at mean-mass temperature of liquid; those without indices taken at local temperature of liquid; 0, flow and heat transfer at $T_w \to T_f$.

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